The Economic Value of Forecasting Left-Tail Risk

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Abstract
We show that it is possible to reduce tail risk without giving up returns. The key is to forecast forward-looking skewness, and by doing it, we are able to identify a sweet spot for a mean-vari- ance-skewness investor. The finding could improve the usefulness of traditional diversification, which typically lowers variance but also results in skewness loss.

Introduction
We can meaningfully reduce future left-tail risk (severe downside events) without giving up expected returns by using relatively simple models for forecasting left-tail risk.

Investors are scared of severe losses, so much so that this aversion allows investors willing to hold stocks, stock options or equity funds the opportunity to earn a “tail-risk premium” (see e.g. Harvey and Siddique 2000; Conrad, Dittmar, and Ghysels 2013; Xiong, Idzorek, and Ibbotson 2014). Anecdotally, among the relatively few practitioners explicitly incorporating higher moments (skewness and kurtosis) into their financial decision-making process, most use historical skewness and kurtosis values as forecasts of skewness and kurtosis. Of course it is future left-tail risk that investors should be concerned with, and in fact we show past tail risk may be a bad predictor for future tail risk. In general, a good time to enter a market to earn the tail-risk premium is after periods of high realized left-tail risk (i.e., buying “when there’s blood in the streets”), while a good time to exit a market is when future or forward-looking left-tail risk is high (a prediction of severe downside losses).

One common measure of left-tail risk is skewness, or more specifically, negative skewness. Negative skewness is often associated with an asymmetric correlation of returns or higher correlation in downward markets (see e.g. Duffee 2000; Cizeau, Potters, and Bouchaud 2001). Forward-looking negative skewness indicates a higher probability of large losses in the future. We forecast forward-looking skewness by examining several factors, including rising and accelerating stock prices (the opposite of past tail risk). Avoiding future left-tail risk is associated with higher returns (Xiong and Ibbotson, 2015). It is the high-priced stock markets that are more susceptible to negative tail risk, whereas stocks that have already dropped in price tend to be more positively skewed.

There is rightfully an enormous body of knowledge around forecasting expected returns¹, given the importance of these figures in investing. Additionally, numerous articles have been

written on forecasting variance and correlation. In comparison, there are only a handful of articles on forecasting skewness. In one of the few articles on the subject, Chen, Hong, and Stein (2001) shows that negative skewness is more likely to occur in individual stocks that: (1) have experienced an increase in trading volume relative to the trend over the prior six months; (2) have increased in value over the prior 36 months; and (3) have large market capitalizations.

Xiong and Ibbotson (2015) finds that accelerated returns (an increase in positive momentum induced perhaps by positive feedback) can forecast negative future skewness for not only individual stocks, but also the aggregate stock market. It shows that realized accelerated returns for individual stocks can lead to poor future performance and a higher likelihood of a future crash or severe loss.

In this paper, we quantify the economic benefit of forecasting skewness for a mean-variance-skewness utility investor, and show how forecasting skewness may enable investors to avoid market crashes. More importantly, these efforts provide insight on diversification, which may not be a “free lunch” because of asymmetric correlation or skewness loss. However, forecasting and thus avoiding future negative skewness can improve diversification in down markets.

**Forecasting Skewness**

We examine the determinants of forward-looking skewness using a series of multiple regression analyses. We initially focus on the U.S. stock market, given its long history, and then look at seven other common equity asset classes, most of which have a much shorter return history than the U.S. stock market. More specifically, we attempt to forecast the skewness for the next six months ($SKEW_{t+6}$), as this could directly serve as an input in a forward-looking optimization designed to find optimal holdings for the next six months.

To predict the forward-looking skewness ($SKEW_{t+6}$), we identify a variety of logical explanatory variables shown in Table 1. To mitigate issues associated with multicollinearity caused by the simultaneous use of a large number of somewhat correlated explanatory variables, we separate the variables into two categories corresponding to two sets of multiple regressions:

| Table 1 Explanatory Variables for Forecasting the Forward-Looking Skewness |
|----------------------------------|----------------------------------|
| **Independent Variables**       | **Description**                  |
| **Technical Factors**            |                                  |
| PAST6SKEW                        | Past 6-Month Skewness            |
| ACC                              | Acceleration of Returns          |
| PAST12RET                        | Past 12-Month Average Return     |
| PAST12STD                        | Past 12-Month Volatility         |
| CHSD                             | Change in Past 12-Month Volatility |
| **Fundamental Factors**          |                                  |
| CAPE                             | Shiller Price-to-Earnings Ratio  |
| CS                               | Credit Spread                    |
| Y1YT                             | One-Year Treasury Yield          |
| NE                               | Equity Share in New Issues       |
a multiple regression based on technical factors and a multiple regression based on fundamental factors. For brevity, we report the regression results for fundamental factors in Appendix A without losing much insight.

For the technical factors, acceleration of returns (ACC) is computed as the geometric return over the most recent six months minus geometric return over the preceding six months. CHSD is the volatility over the most recent six months minus the volatility over the preceding six months. Notice that all of these technical factors are calculated based on daily returns, and then for all technical factors except PAST6SKEW, using a conversion that is analogous to annualizing summary statistics, we convert daily statistics into monthly frequency to facilitate the interpretation of regression results. Given that these technical factors are all derived from the series in question, the factors go back as far the series being analyzed. For computational efficiency, we fix 21 (252) trading days as one month (year), although the number of typical trading days in a month (year) has evolved over time.

For all regressions performed in this paper, we use daily data with quarterly overlapping observations in order to add statistical power. In other words, the regression is performed on a quarterly (63 trading days) rolling basis to forecast skewness over the next six months (126 trading days). Additionally, the reported t-statistics are adjusted for serial correlation and heteroskedasticity using the method described in Newey and West (1987).

**U.S. Stock Market**

For the U.S. stock market, we used daily return data for the CRSP Value-Weighted Stock Index Total Return series downloaded from the French Data Library, covering 88.5 calendar years from July 1926 to December 2014. Table 2 shows the regression results for forward-looking six-month skewness as well as the adjusted t-statistics in parentheses. Regression results for forecasting other horizons, such as the forward-looking 3-, 9-, 12-, 24-, or 36-month skewness are largely unchanged (results are not reported).

In unreported results, we simultaneously used all of the technical and fundamental factors together in one multiple regression analysis and found that the results are consistent with the two separate sets of results reported here. Other ways to measure the acceleration, such as an exponential weighting scheme used in Xiong and Ibbotson (2015), can yield more powerful regression results. By using fixed 21 trading days as one month, we can mitigate some calendar month effects, such as the December or January effect. However, using calendar month yields similar results.

Table 2 Forward-Looking 6-Month Skewness Regression Results Against the Technical Factors for the Daily U.S. Stock Market Returns from July 1926 – December 2014*

<table>
<thead>
<tr>
<th></th>
<th>Intercept</th>
<th>PAST6SKEW</th>
<th>ACC</th>
<th>PAST12RET</th>
<th>CHSD</th>
<th>PAST12STD</th>
</tr>
</thead>
<tbody>
<tr>
<td>SKEW</td>
<td>−0.34</td>
<td>0.08</td>
<td>−2.02</td>
<td>−14.99</td>
<td>−1.89</td>
<td>4.12</td>
</tr>
<tr>
<td></td>
<td>(−3.18)</td>
<td>(1.04)</td>
<td>(−2.20)</td>
<td>(−5.55)</td>
<td>(−0.91)</td>
<td>(1.94)</td>
</tr>
</tbody>
</table>

* Quarterly overlapping observations are used in the time-series regression. T-statistics are in parentheses and adjusted for serial correlation and heteroskedasticity.

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2 In unreported results, we simultaneously used all of the technical and fundamental factors together in one multiple regression analysis and found that the results are consistent with the two separate sets of results reported here.

3 Other ways to measure the acceleration, such as an exponential weighting scheme used in Xiong and Ibbotson (2015), can yield more powerful regression results.

4 By using fixed 21 trading days as one month, we can mitigate some calendar month effects, such as the December or January effect. However, using calendar month yields similar results.

5 http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html. Total return is derived from the first of the three Fama-French factors plus cash.
In Table 2, we see a negative intercept with a highly significant t-statistic in the first column, indicating that the U.S. stock market tends to have negative skewness on average. We will discuss the adjusted R-squared value shortly.

Focusing on the explanatory variables, positive acceleration (ACC) and positive past 12-month return (PAST12RET) predict future negative skewness (hence the negative coefficients), with acceleration statistically significant at the 5 percent level and PAST12RET statistically significant at the 1 percent level. More precisely, on average, a 1 percent monthly acceleration decreases future skewness by 0.028 (i.e., more severe expected negative skewness); and on average, a 1 percent increase of past 12-month average return decreases future skewness by 0.14 (i.e., more severe expected negative skewness).\(^6\)

Change in standard deviation (CHSD) is not significant, but past volatility (PAST12STD) is nearly significant at the 5 percent level. The positive PAST12STD coefficient indicates that a low (high) volatility forecasts a future negative (positive) skewness, which suggests a trade-off between volatility and skewness. Strategies that buy when volatility is low (such as low-volatility or target volatility) might be susceptible to crash risk or have negative skewness. On the other hand, more positive skewness is preferred but it may come at the expense of higher volatility. From a practical perspective, low-volatility and target volatility may look relatively good when one focuses on “normal” measures of risk/performance and relatively worse when one uses risk measures that capture non-normal qualities, such as conditional value-at-risk.

Note that the second column of Table 2 shows that past skewness is not statistically significant in forecasting future skewness (t-stat = 1.05) in the multiple regression. This is inconsistent with the results shown in Chen, Hong, and Stein (2000) and Xiong and Ibbotson (2015) where both studies found that past skewness significantly forecasts future skewness in cross-sectional regressions for individual stocks. These past results are unique to specific time periods studied. We employ an extended subperiod analysis to shed some light on this apparent inconsistency.

Figure 1 shows how the t-stats for the regression coefficients of the three independent variables (PAST6SKEW, ACC, and PAST12RET) in forecasting next six-month skewness changed over time from July 1927 to December 2014. The multiple regressions were performed on a 10-year rolling basis, and each regression covers a 20-year period, except the last data point which covers only 12 years, from 2002 to 2014. Figure 1 indicates that past skewness is not stable in forecasting future skewness, and positive coefficients appear only from the 1960s to the 1990s. On the other hand, t-stats for both ACC and PAST12RET are more

\(^6\) We also forecast the first difference of skewness, i.e., the change in skewness \(\text{SKEW}_{t+6} - \text{SKEW}_t\) for the U.S. market. In unreported results, only ACC is overwhelmingly significant at the 1% level in forecasting the first difference of forward-looking skewness.
Persistently negative over time at an absolute value that is close to or exceeds 1.99 (significant at the 5 percent level) through time.  

**Figure 1** Time-Series T-Statistics for the Regression Coefficients (PAST6SKEW, ACC and PAST12RET) in Forecasting Future Six-Month Skewness *  

Furthermore, our analyses indicate that past skewness tends to persist only in less volatile markets (t-stat = 3.13), but the predictive power disappears in more volatile markets (t-stat = −0.28). The period from 1960 to 2000 appeared to be less volatile than periods before 1960 or after 2000, which contributed to a significant t-stat for the past skewness in earlier studies. We conclude that in the absence of a regime-switching model conditional on volatility, past skewness is not a good predictor for future skewness.

Returning to the robustness of the predictive power of our regression coefficient in Table 2, we compute the out-of-sample adjusted R-squared in the spirit of Welch and Goyal (2008). The adjusted R-squared for the (in-sample) regression shown in Table 2 was 16.38%. The out-of-sample adjusted R-squared obtained by using all five technical variables shown in Table 2 was 10.54%. Interestingly, the out-of-sample adjusted R-squared found by using only ACC and PAST12RET as independent variables, has a higher value of 11.87%. It confirms the predictive power of ACC and PAST12RET shown in Figure 1. Bootstrap analysis shows that the t-stat for the adjusted R-squared is significant at the 1 percent level for both in-sample and the two out-of-sample forecasts.

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7 The time-series t-stat for the regression coefficient, computed as the mean divided by standard error, is significant for ACC and PAST12RET at the 5% level, but not significant for PAST6SKEW.

8 The less volatile (more volatile) market is defined as daily volatility over the past 12 months being below (above) its average value of 0.95%.

9 The out-of-sample forecasted skewness was tested against the historical mean skewness in an expanding window. The first 10-years of data were used for the first out-of-sample estimate. Forecasted error is measured as the mean squared difference between forecasted skewness (or historical mean skewness for benchmark) and realized skewness.
In short, the acceleration of returns (ACC) and PAST12RET appear to have robust power in predicting forward-looking skewness for the U.S. stock market. Again, past skewness is not a reliable predictor for future skewness, and appears to be conditional on the volatility regime. Low volatility strategies tend to crash or have negative future skewness.

**Seven Equity Asset Classes**

We now broaden our analysis to include seven commonly used equity asset classes. Table 3 identifies the seven asset classes, their respective index proxies, and the date range. Here we focus on daily data obtained from Morningstar Direct.

### Table 3 Seven Equity Asset Classes

<table>
<thead>
<tr>
<th>Asset Class</th>
<th>Index</th>
<th>Daily Data Coverage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Large Growth (LG)</td>
<td>Russell 1000 Growth</td>
<td>Jan 1991–Dec 2014</td>
</tr>
<tr>
<td>Large Value (LV)</td>
<td>Russell 1000 Growth</td>
<td>Jan 1991–Dec 2014</td>
</tr>
<tr>
<td>REITs</td>
<td>FTSE NAREIT Equity REITs</td>
<td>Jan 1999–Dec 2014</td>
</tr>
<tr>
<td>Non-U.S. Dev. Equity (EAFE)</td>
<td>MSCI EAFE</td>
<td>Jan 2000–Dec 2014</td>
</tr>
</tbody>
</table>

As we did for the U.S. equity market, we attempt to understand which factors explain skewness for the seven equity asset classes. Table 4 reports multiple regression results for the technical factors, while Table A2 in Appendix A reports results for the fundamental factors.

### Table 4 Forward-Looking Six-Month Skewness Regression Results for Technical Factors* in the Seven Equity Asset Classes

<table>
<thead>
<tr>
<th>Asset Class</th>
<th>SKEWt+6</th>
<th>Intercept</th>
<th>PAST6SKEW</th>
<th>ACC</th>
<th>PAST12RET</th>
<th>CHSD</th>
<th>PAST12STD</th>
<th>Adjusted R²</th>
</tr>
</thead>
<tbody>
<tr>
<td>Large Growth (LG)</td>
<td>–0.37</td>
<td>–0.02</td>
<td>–4.73</td>
<td>–6.07</td>
<td>–3.60</td>
<td>5.16</td>
<td>18.30%</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(–2.02)</td>
<td>(–0.12)</td>
<td>(–1.20)</td>
<td>(–1.92)</td>
<td>(–2.08)</td>
<td>(1.63)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Large Value (LV)</td>
<td>0.01</td>
<td>–0.17</td>
<td>–4.59</td>
<td>–17.88</td>
<td>–5.08</td>
<td>–2.08</td>
<td>13.73%</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1.54)</td>
<td>(–1.92)</td>
<td>(–1.20)</td>
<td>(–1.92)</td>
<td>(–0.79)</td>
<td>(–0.45)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Small Growth (SG)</td>
<td>–0.97</td>
<td>–0.04</td>
<td>–3.67</td>
<td>3.65</td>
<td>0.92</td>
<td>10.55</td>
<td>25.08%</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(–2.37)</td>
<td>(–0.24)</td>
<td>(–2.57)</td>
<td>(1.20)</td>
<td>(0.41)</td>
<td>(2.76)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Small Value (SV)</td>
<td>–0.41</td>
<td>0.12</td>
<td>–3.33</td>
<td>–5.46</td>
<td>0.06</td>
<td>4.42</td>
<td>13.83%</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1.09)</td>
<td>(1.96)</td>
<td>(–1.06)</td>
<td>(–0.02)</td>
<td>(1.26)</td>
<td>(1.64)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>REITs</td>
<td>–0.15</td>
<td>–0.08</td>
<td>–1.59</td>
<td>–7.25</td>
<td>–1.13</td>
<td>0.95</td>
<td>1.89%</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(–1.06)</td>
<td>(–1.11)</td>
<td>(–1.11)</td>
<td>(–1.50)</td>
<td>(0.64)</td>
<td>(0.84)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>EAFE</td>
<td>–0.25</td>
<td>–0.05</td>
<td>–6.12</td>
<td>–6.56</td>
<td>–4.46</td>
<td>2.77</td>
<td>6.64%</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(–1.22)</td>
<td>(–0.35)</td>
<td>(–1.37)</td>
<td>(–1.51)</td>
<td>(–1.73)</td>
<td>(0.84)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>EM</td>
<td>–0.42</td>
<td>0.04</td>
<td>–2.99</td>
<td>–6.56</td>
<td>–2.01</td>
<td>4.34</td>
<td>23.09%</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(–2.25)</td>
<td>(–2.69)</td>
<td>(–1.03)</td>
<td>(–1.26)</td>
<td>(1.63)</td>
<td>(1.63)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* Technical factors are asset-specific. Quarterly overlapping observations are used in the time-series regression. t-Statistics are in parentheses and adjusted for serial correlation and heteroskedasticity.
Table 4 contains a wealth of information. As expected, the coefficients and the corresponding robust t-statistics vary per asset class. For four of the seven asset classes, we arrive at intercepts that are statistically significant. Looking at the independent variables, the past skewness (PAST6SKEW) is again not a good predictor for future skewness for all asset classes. Acceleration (ACC) is the most significant predictor of future negative skewness (five t-stats are significant at the 5 percent level out of seven asset classes), followed by PAST12RET (two asset classes have significant t-stats). On average, the rise of volatility (CHSD) has negative impact on skewness, but it is only significant for Large Value and EAFE at the 10 percent level. On average, the level of volatility (PAST12STD) has positive impact on skewness, and it is significant for Small Growth at the 5 percent level.

Overall, Table 4 is consistent with Table 2 even though Table 4 has a much shorter history. Acceleration remains a robust indicator for negative skewness for the equity asset classes. PAST12RET has on average negative impact on skewness but has slightly less explanatory power than ACC for equity asset classes over the last two decades. Other factors have either less power or mixed results for explaining skewness for the seven equity asset classes.

Quantifying the Benefit of Forecasting Skewness
In the previous section, we developed models for forecasting the skewness of U.S. equities and seven additional equity asset classes over the next six months. In this section, we begin with the thesis that we can meaningfully reduce future tail risk (negative skewness) in an investment portfolio if we can successfully forecast future skewness. If we accept that returns are difficult to forecast, perhaps instead tail risk can be reduced without giving up returns? To answer this important question, we need to quantify the impact of forecasting skewness on portfolio performance and any increase in utility.

We forecast skewness using the regression results shown in Table 2, i.e.:

\[
SKEW_{t+6} = -0.34 + 0.08 \cdot \text{PAST6SKEW}_t - 2.82 \cdot \text{ACC}_t - 14.09 \cdot \text{PAST12RET}_t - 1.89 \cdot \text{CHSD}_t + 4.12 \cdot \text{PAST12PSD}_t 
\]

Starting in July 1927, with a quarterly time step, we do the following four steps:

1. Forecast next six-month skewness based on Equation 1;

2. For each forecasted skewness, compute the arithmetic average return, standard deviation or volatility, skewness, and utility (discussed shortly) for the ensuing realized six months of daily returns, and record it as a pair of (forecast6Skew) and (future6Ret, future6SD, future6Skew, future6Utility);

10 We performed a pooled regression and a panel regression with fixed effects, and both regressions confirmed that both ACC and PAST12RET remain significant at the 1% level.
3. Sort the forecasted skewness (forecast6Skew) from negative to positive into 20 bins, where each of the 20 bins contains 18 pairs of (forecast6Skew) and their corresponding realized (future6Ret, future6SD, future6Skew, future6Utility).  

4. For each of the 20 bins, compute the arithmetic average of 18 pairs of (forecast6Skew) and (future6Ret, future6SD, future6Skew, future6Utility). The results are shown in Figure 2 and 3.

The horizontal axis in Figure 2 represents the average forecasted skewness (forecast6Skew) for the next six months for each of the 20 bins. Then, for each of the 20 bin portfolios we plot three averages: average realized skewness (future6Skew), average realized return (future6Ret), and average realized standard deviation (future6SD). The right-hand vertical axis corresponds to the average realized skewness (green triangles) and the left-hand vertical axis corresponds to realized return (blue diamonds) and realized volatility (red squares), respectively.

Starting with skewness, the twenty green triangles and the fitted green line, show the power of Equation (1) in predicting future skewness. The slope is significant at the 1 percent level with t-stat = 8.25.

40 Bin Portfolios vs. Average Predicted Skewness from t to (t+6)

Moving to return, on average, the future six-month returns (future6Ret) are slightly higher when forecasted skewness is more negative (i.e., past 12-month returns are positive) than the

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11 There are 23,385 trading days from 7/1/1926 to 12/31/2014, corresponding to 92 fixed-trading years (i.e. 252 trading days in one year) and 88 calendar years. There are on average 25 trading days in a calendar month before 1945, and about 21 trading days in a month since then.

12 We constructed 40 bin portfolios in similar manner which gave similar results.
future six-month returns when forecasted skewness is more positive (i.e. past 12-month returns are negative), which is somewhat consistent with the time-series momentum documented by (Moskowitz, Ooi, and Pedersen 2012). It is well-known that momentum strategies tend to have negative skewness.

Finally, moving to standard deviation, future volatility (future6SD) is higher when forecasted skewness is very positive, which suggests a trade-off between volatility and skewness as mentioned above.

In Figure 3 we look at the realized future utility (future6Utility) that an investor would have derived from the 20 bin portfolios. Figure 3 represents the average realized future utility for the 18 sets of six months’ of daily returns for each bin. In Figure 3 we measure the utility of the 20 bin portfolios using three utility functions: 1) the traditional mean-variance utility function; 2) a mean-skewness utility function (i.e., for lottery-type investors who seek huge upside potential and are willing to sacrifice Sharpe ratio or mean-variance utility to gain higher skewness, see Ilmanen, 2012); and 3) a mean-variance-skewness utility function.

Figure 3 U.S. Stock Market: Average Future Six-Month Utilities (Mean-Variance, Mean-Variance-Skewness, and Mean-Skewness) for the 20 Bin Portfolios vs. Average Predicted Rescaled Skewness from t to (t+6)*

*Solid line is the smoothed curve for the mean-variance-skewness utility.
The mean-variance-skewness utility function incorporates mean, variance and skewness. It takes the following form (see e.g. Harvey and Siddique 2000; and Briec, Kerstens, and Jokung 2007):

\[ U = \mu - \lambda V + \kappa S \]  

(2)

where \( \mu \), \( V \), and \( S \) are mean return, variance, and skewness, respectively, \( \lambda \) and \( \kappa \) are risk-aversion parameters for variance and skewness, respectively. With Equation (2), utility is improved by increases in return and skewness and decreases in variance. Note that in the special case, when \( \kappa = 0 \), it simplifies to the traditional mean-variance utility. Similarly, when \( \lambda = 0 \), Equation (2) simplifies to the mean-skewness utility function. Many investors care about skewness. Prudent investors prefer positive skewness because it implies a low probability of experiencing a large negative return and a high probability of a large positive return. Lottery players may be risk-averse but they favor positive skewness of returns—the potential for a huge gain.

The parameters \( \lambda \) and \( \kappa \) in Equation (2) are fixed to be 5 and 0.2, respectively. They correspond to typical conservative and prudent investors. However, our conclusions remain unchanged for a relatively wide range of \( \lambda \) and \( \kappa \).

The highest mean-variance-skewness utility appears to be realized when the predicted future skewness is in the middle range. For example, the average mean-variance-skewness utility from Bin 9 to Bin 15 is \(-0.01\), significantly higher than the average of the other 13 bins \((-0.15)\) at the 1 percent level. Note that Bin 1 has the most negative forecasted skewness in Figure 3 (left side). Therefore, the mean-variance-skewness frontier created by fitting a polynomial to the 20 bin portfolios in Figure 3 clearly shows a sweet spot. The sweet spot is approximately centered at a forecasted skewness of \(-0.38\). The sweet spot implies that one can lower downside risk without giving up returns by forecasting skewness and by lowering equity market exposure when predicted future skewness is extremely negative or when past returns have been rising and accelerating.

For the traditional mean-variance investor, the utility curve is similar to the mean-variance-skewness utility curve, but slightly higher (lower) on the left (right) side because skewness is more (less) negative on the left (right) side.

For a lottery-type investor, represented by the mean-skewness utility, the best timing appears to be when the forecasted skewness is very positive—a potential for a large positive return after a market has suffered from a huge loss.

\[ \lambda \] is selected to be 5 for a conservative investor (see Sharpe, Chen, Pinto, and McLeavey, (2007)). \( \kappa \) is selected to be 0.2 so that it will make the skewness preference (the third term in Equation 2) comparable to the variance aversion (the second term in Equation 2).
In summary, we have shown that for the mean-variance-skewness utility, lowering downside risk without giving up returns is achievable, and the key is to forecast skewness and avoid very elevated and accelerated returns.

**Robustness Check**

As a robustness check, we repeat the same analyses for the seven equity asset classes shown in Figures 4 and 5, which correspond to Figures 2 and 3 for the U.S. stock market, respectively. Because we have relatively short histories for all of the seven asset classes, we pool them so that we have 25 six-month daily returns for each of the 20 bin portfolios. The pooling provides cross-sectional information even though the seven equity classes are highly correlated. For this robustness check, the skewness forecast is now based on a pooled regression as:

\[
\text{SKEW}_{t+6} = -0.31 + 0.01 \cdot \text{PAST6SKEW}_t - 3.52 \cdot \text{ACC}_t - 5.59 \cdot \text{PAST12RET}_t - 1.66 \cdot \text{CHSD}_t + 2.94 \cdot \text{PAST12PSD}_t
\]

The utility function in Figure 5 is the same as the one in Figure 3.

**Figure 4** Pooled Seven Equity Asset Classes: Realized Average Future Six-Month Mean Returns, Volatilities, and Skewness for the 20 Bin Portfolios vs. Average Predicted Skewness from \(t\) to \((t+6)\)
Figures 4 and 5 are consistent with Figures 2 and 3, respectively. As seen in Figure 4, skewness remains highly predictable (slope has a t-stat of 6.79). In Figure 5, the average mean-variance-skewness utility for the sweet spot (from Bin 8 to Bin 14) is –0.06, significantly higher than the average of the other 13 bins (–0.24) at the 1 percent level. The sweet spot is located in a similar position for both Figures 3 and 5. The sweet spot in Figure 5 is approximately centered when the forecasted skewness equals –0.27.

For lottery-type investors with a mean-skewness utility function, the best time to invest once again appears to be when forecasted skewness is very positive (i.e., after the market has suffered from a huge loss).

**Implication for Diversification**

Forecasting skewness can further shed light on diversification, which has been considered to be a “free lunch” in investing in that it allows an investor to get greater return without taking more risk. It is well known that traditional diversification lowers variance, but also fails to protect investors to the degree that one would infer if returns were normally distributed during a severe down market. This difference is due to non-normal returns and asymmetric correlation (see Chua, Kritzman and Page 2009). The observation that increased diversification leads to skewness loss has been documented for quite some time, for example, Kraus and Litzenberger (1976), and Simkowitz and Beedles (1978). One can reconcile these two lines of research with the fact that asymmetric correlation and negative skewness are closely related (e.g. Duffee 2000; Cizeau, Potters, and Bouchaud, 2001).
Therefore the failure of diversification to protect on the downside is directly related to skewness loss or asymmetric correlation. By forecasting skewness for each asset in a portfolio, one can hopefully reduce the portfolio’s skewness loss by avoiding or under-weighting assets with extremely elevated, accelerated, and thus unsustainable returns (the left side of Figure 2). On the other hand, for a lottery-type investor, searching for a very positive forward-looking skewness goes hand-in-hand with an increased volatility (the right side of Figure 2). Depending on an investor’s utility preference, our findings provide important implications for optimal portfolio construction that is left for future research.

Conclusion
To many investors, minimizing future downside or tail risk without giving up return should be one of the most important goals in portfolio construction. The key is to accurately forecast forward-looking left-tail risk, in particular, negative skewness (i.e., a high probability of large losses).

We identify factors that can forecast negative skewness for the U.S. stock market and seven other equity asset classes. We find that acceleration of returns and past 12-month returns have robust predictive power in forecasting negative skewness for equity asset classes. We also find that past skewness is not a reliable predictor for future skewness, and that low-volatility strategies tend to have negative skewness.

We show that a sweet spot (lowering future tail risk without giving up returns) for a mean-variance-skewness investor exists when forecasted skewness is in the middle range, i.e. when past returns and accelerations are not extremely positive or negative. For a lottery-type investor, the best timing for market entry is when forecasted skewness is very positive or after market has suffered from a huge loss.

Because the failure of diversification is directly related to skewness loss or asymmetric correlation, one may be able to reduce skewness loss by forecasting skewness and thus improve diversification by underweighting those unsustainably accelerated assets.
Appendix A. Forecasting Skewness by Using Fundamental Factors

As shown in Table 1, we use four fundamental factors to forecast skewness.\(^{14}\) The Shiller PE (CAPE) ratio represents what an investor pays (current price) for the last 10 years’ average real S&P 500 index earnings. It is well documented that a relatively high Shiller PE is followed by lower future realized returns. Credit spread (CS) and changes in credit spread (CHCS) are important variables for understanding market stress.\(^{15}\) The one-year Treasury yield (Y1YT) is a standard variable thought to explain subsequent returns. Our final fundamental factor is equity share in new issues (NE)—which is defined in Baker and Wurgler (2000) as the ratio of new equity share over the total of new equity share and new debt share.\(^{16}\) High NE values forecast low stock market returns. NE is a proxy for investor sentiment, which is typically extremely positive prior to market crashes.

All four fundamental factors are averaged over the last 12 months as independent variables in the regressions to reflect the levels of those variables. In addition to the variable itself, we add the changes of these independent variables: change in CAPE (CHCAPE), change in credit spread (CHCS), change in one-year Treasury yield (CHY1YT), and change in new equity share (CHNE). They are defined as the difference between the average of first six-month values and the average of second six-month values (e.g., CHCAPE = most recent six-month average CAPE minus preceding six-month average CAPE).

The technical factors are asset class-specific given that they are based on their own historical returns, whereas the selected fundamental factors are used across all of the asset classes.

For fundamental factors, we shortened the analysis period to January 1980 to December 2014 due to data limitations. Table A1 shows the regression results on future skewness against the fundamental factors. The adjusted R-squared for the regression was 18.03%.

\(^{14}\) We tested a few other fundamental factors, including term spread, unemployment rate, inflation, market cap-to-GDP ratio, etc., and found these factors are overall less significant and have mixed roles in forecasting skewness. They are not reported for brevity.

\(^{15}\) Credit spread is calculated as the difference of the Barclays US Corp Baa Yld index and Barclays US Corp Aaa Yld index.

\(^{16}\) Like Baker and Wurgler (2000), we use the Federal Reserve bulletin data as the primary source. These data series from the Federal Reserve are gross totals of equity and debt share and do not subtract repurchases or debt retirements.

---

### Table 1A Forward-Looking Six-Month Skewness Regression Results Against the Fundamental Factors for the Daily U.S. Stock Market Returns from January 1980 – December 2014*  

<table>
<thead>
<tr>
<th></th>
<th>Intercept</th>
<th>CHCAPE</th>
<th>CAPE</th>
<th>CHCS</th>
<th>CS</th>
<th>CHY1YT</th>
<th>Y1YT</th>
<th>CHNE</th>
<th>NE</th>
</tr>
</thead>
<tbody>
<tr>
<td>SKEW(_{t+6})</td>
<td>-0.99</td>
<td>-0.11</td>
<td>0.02</td>
<td>-0.16</td>
<td>0.06</td>
<td>-0.04</td>
<td>0.00</td>
<td>-1.81</td>
<td>2.06</td>
</tr>
<tr>
<td></td>
<td>(-3.17)</td>
<td>(-2.12)</td>
<td>(1.92)</td>
<td>(-0.84)</td>
<td>(0.33)</td>
<td>(-0.66)</td>
<td>(0.16)</td>
<td>(-1.65)</td>
<td>(1.92)</td>
</tr>
</tbody>
</table>

* Quarterly overlapping observations are used in the time-series regression. T-statistics are in parentheses and adjusted for serial correlation and heteroskedasticity.
From Table A1, we continue to see a negative intercept with a highly significant robust t-statistic. Only CHCAPE is significant at the 5 percent level, indicating that a rise in CAPE negatively impacts the forward-looking skewness. CHNE is significant at the 10 percent level. This is intuitive, as a rise in stock price relative to earnings signals an overvalued stock market (CHCAPE is 80 percent correlated with PAST12RET for U.S. stocks), and the increase of equity share in new issues indicates potential overconfidence. Both conditions can lead to a market reversal, hence negative forward-looking skewness. Both CAPE and NE have a positive impact on forward-looking skewness, which is somewhat surprising (we would expect a negative coefficient for both of them). The change in credit spreads (CHCS) coefficient is negative but not significant, suggesting that an increase of credit spread contributes to a more negative forward-looking skewness.

Table A2 shows regression results on forward-looking skewness against the fundamental factors for the seven equity asset classes. One caveat is that all of the interpretations should be treated carefully given the relatively short history of daily returns. In general, the rise of CAPE (CHCAPE) has a negative impact while CAPE has a positive impact on the skewness for the seven equity asset classes. CHCAPE is significant at the 1 percent level for Large Growth and Small Growth. CAPE is significant at the 1 percent for Large Growth, Small Growth, and Small Value.

<table>
<thead>
<tr>
<th>Intercept</th>
<th>CHPE</th>
<th>PE</th>
<th>CHCS</th>
<th>CS</th>
<th>CHY1YT</th>
<th>Y1YT</th>
<th>CHNE</th>
<th>NE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Large Growth</td>
<td>-0.99</td>
<td>-0.06</td>
<td>0.02</td>
<td>-0.06</td>
<td>0.23</td>
<td>0.10</td>
<td>0.01</td>
<td>0.46</td>
</tr>
<tr>
<td>Large Value</td>
<td>-0.05</td>
<td>0.01</td>
<td>0.01</td>
<td>0.30</td>
<td>0.37</td>
<td>0.14</td>
<td>0.02</td>
<td>2.16</td>
</tr>
<tr>
<td>Small Growth</td>
<td>-0.78</td>
<td>-0.07</td>
<td>0.03</td>
<td>0.22</td>
<td>0.34</td>
<td>0.20</td>
<td>0.02</td>
<td>2.13</td>
</tr>
<tr>
<td>Small Value</td>
<td>-0.96</td>
<td>-0.05</td>
<td>0.03</td>
<td>0.06</td>
<td>0.23</td>
<td>0.03</td>
<td>-0.10</td>
<td>1.77</td>
</tr>
<tr>
<td>REITs</td>
<td>-0.23</td>
<td>0.06</td>
<td>-0.03</td>
<td>0.33</td>
<td>0.34</td>
<td>0.02</td>
<td>0.15</td>
<td>2.20</td>
</tr>
<tr>
<td>EAFE</td>
<td>-0.38</td>
<td>-0.01</td>
<td>0.00</td>
<td>0.17</td>
<td>0.17</td>
<td>-0.04</td>
<td>0.03</td>
<td>2.76</td>
</tr>
<tr>
<td>EM</td>
<td>-0.66</td>
<td>0.00</td>
<td>0.02</td>
<td>0.17</td>
<td>0.06</td>
<td>-0.24</td>
<td>-0.12</td>
<td>0.48</td>
</tr>
</tbody>
</table>

* Fundamental factors are common to all asset classes. Quarterly overlapping observations are used in the time-series regression. T-statistics are in parentheses and adjusted for serial correlation and heteroskedasticity.

The other independent variables have mixed results in explaining skewness for the seven asset classes. The rise of one-year Treasury yield (CHY1YT) has a positive impact on skewness, except for Non-U.S. Dev. Equity and Emerging Markets. The level of one-year Treasury bill yield (Y1YT) has significant positive impact on skewness for REITs, and has significant negative impact for Small Growth, Small Value, and Emerging Markets.
References


